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PREFACE

This Report contains material prepared for the I.A.U. Symposium No. 21 on the System of Astronomical Constants.

ABSTRACT

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This Report is concerned with the author's work on the radar determination of the astronomical unit (Refs. 1, 2, 3) and with the significance of current radar observations on the system of astronomical constants. It is limited principally to that part of the system which is obviously affected by the radar observations. Consequently, the discussion pertaining to the geodetic constants of the Earth, for example, is also limited.

The Report discusses the work of Newcomb and de Sitter from the standpoint of the definitions of the fundamental constants and the theoretical relationships between them. These theoretical relationships are applied to the radar results when possible. The main body of the Report is devoted to a brief but exact discussion of the determination of the astronomical unit with radar and to an extensive error analysis of the technique.

Because of the many theoretical relationships between the constants, a certain group of them have been selected (primarily by Newcomb and de Sitter) as fundamental. The discussion suggests that this particular division of the constants may be profitably revised because of the inclusion of radar measurements of distances and velocities to the observational material of dynamical astronomy. AUTHOR

I. THE SYSTEM OF ASTRONOMICAL CONSTANTS

The fundamental constants of the Earth consisting of the elements of its orbit; its mass; constants specifying size, shape, orientation, rotation, and inner constitution; and the velocity of light comprise the system of astronomical constants. The group of constants has been called a system because it comprises a model of the Earth and its motions. The interpretation of all astronomical observations depends on this particular model. Furthermore, because of the many theoretical relationships involving several of the fundamental constants, some of these are necessarily related systematically.

The system of astronomical constants has apparently evolved from Newcomb's work reported in Vols. I and II of the *Astronomical Papers* and his *Astronomical Constants* (Ref. 4), which is an exhaustive treatment of the subject as well as a compilation of important formulas. These works, particularly the latter, served as the basis for the system of constants adopted by the Paris conference in 1896. Many of the adopted values were integrated into Newcomb's tables of the Sun and the four inner planets. Partly because of the fundamental importance of these tables, astronomers since Newcomb have been reluctant to change the values of the constants even though several important inconsistencies are known to exist in the system.

II. RELATIONSHIPS BETWEEN THE ASTRONOMICAL CONSTANTS

De Sitter (Ref. 5) attempted to construct a rigorous system of astronomical constants based on the observations available up to 1938. In so doing, he set down a series of relationships and ideas which still serve as a guide for a rediscussion of the fundamental constants. The relationships that appear to be important from the standpoint of interpreting the astronomical unit (AU) as determined from radar measurements will be written with little development and will be applied to the numerical results later in this Report.

De Sitter selected eight constants as "fundamental":

1. $\pi_\odot$ solar parallax
2. μ^{-1} Moon–Earth mass ratio
3. c velocity of light
4. $(C-A)/C$ dynamic compression
5. R_1 *mean* radius of the Earth
6. g_1 gravity acceleration at mean latitude
7. k small constant relating to the Earth's interior
8. λ_1 small constant relating to the Earth's interior

All of the remaining constants are then considered as derived constants. By the use of R_1 and g_1, the relationships of geodesy are supposedly simplified since R_1 is defined as the radius on an ellipsoidal Earth at a latitude $\phi = \sin^{-1}\sqrt{1/3}$, where the mass of the Earth acts as a point mass at the center of the Earth. The equatorial radius of the Earth, b, is then given by

$$b = R_1\left(1 + \frac{1}{3}\epsilon - \frac{4}{9}\epsilon^2 + \frac{8}{9}k\right) \tag{1}$$

where ϵ is the elliptical flattening of the Earth. De Sitter then defined the ratio of centrifugal force to gravitational force as

$$\rho_1 = \frac{\omega^2 R_1^2}{fM_1} \tag{2}$$

where ω is the angular velocity of the Earth, f the gravitational constant, and M_1 the mass of the Earth. Then it can be shown that

$$g_1 = \frac{fM_1}{R_1^2}\left(1 - \frac{2}{3}\rho_1 + \frac{14}{9}\epsilon^2 - \frac{10}{9}\epsilon\rho_1 - \frac{16}{9}k\right) \tag{3}$$

De Sitter adopted the values

$$\begin{aligned}
R_1 &= 6{,}371{,}260\,(1+u),\ \text{m}\\
g_1 &= 979.770\,(1+v),\ \text{cm/sec}^2\\
H &= (C-A)/C = 0.003279423\,(1+w)\\
k &= 0.000\,00050 + 10^{-3}\,X\\
\lambda_1 &= 0.000\,40 + \psi\\
\pi_\odot &= 8''.8030\,(1+x)\\
C &= 299{,}774\,(1+y),\ \text{km/sec}\\
\mu^{-1} &= 81.53\,(1+z)
\end{aligned}$$

The AU in kilometers is *defined* by the relationship

$$1\ \text{AU} = \frac{b}{\pi_\odot \sin 1''} \tag{4}$$

which, from Eq. (1), yields

$$1\ \text{AU} = \frac{R_1}{\pi_\odot \sin 1''}\left(1 + \frac{1}{5}\epsilon - \frac{4}{9}\epsilon^2 + \frac{8}{9}k\right) \tag{5}$$

His *derived* AU then becomes

$$1\ \text{AU} = 149{,}453{,}000\ \text{km}$$

$$[1 - x + 1.0002u - 0.0002v + 0.0009w + 0.0007X + 0.009\psi]$$

The following series of definitions from de Sitter will be used later in this Report. All of the relationships are standard and require no discussion. From Kepler's law, the semimajor axis a_0, in astronomical units, is defined by

$$n^2 a_0^3 = k^2(1 + m) \tag{6}$$

where n is the mean motion of the Earth, k is gauss' constant, and m is the ratio of the mass of the Earth plus Moon to the mass of the Sun. The constant of aberration k is *defined* by

$$k = \frac{n\, a \sec\phi}{86400\, C}$$

$$k = \frac{n\, b \sec\phi}{84600\, \pi_\odot \sin 1''e} \tag{7}$$

and the light time for 1 AU, τ, is *defined* by

$$\tau = \frac{b}{c\, \pi_\odot \sin 1''}, \text{ sec} \tag{8}$$

Now, using Eqs. (1), (3), (4), (5), and (6), and after considerable manipulation,

$$\frac{1+m}{m}\pi_\odot^3 = \frac{n^2 R_1 (1+\nu_1)^3}{g_1 (1+\mu)(86400)^2 \sin 1''}$$

$$\times\left(1 - \nu_3 + \epsilon - \frac{2}{3}\rho_1 + \frac{5}{9}\epsilon^2 - \frac{16}{9}\epsilon\,\rho_1 + \frac{8}{9}k\right) \tag{9}$$

where ν_3 is the fraction of the Earth mass that must be added to M_1 to include the mass of the atmosphere. Equation (9) can be considered as the major relationship given by de Sitter since it relates the mass of the Earth–Moon system to the fundamental constants $\pi_\odot$, R_1, μ, and g_1. This expression has been employed by investigators to determine the constants from the motion of Eros in particular (see Sect. III).

The parallactic inequality is the term $-P \sin D$ in the Moon's ecliptic longitude. The value of P is given from Brown's lunar theory,

$$P = 49853''.2 \frac{1-\mu}{1+\mu} \frac{\pi_\odot}{\sin \pi_{☾}} \tag{10}$$

and finally, for the constant of the lunar inequality, de Sitter introduces

$$L = \frac{\mu}{1+\mu} \frac{\pi_\odot}{\sin \pi_{☾}} \tag{11}$$

The last two expressions are important relationships between μ, $\pi_\odot$, and $\pi_{☾}$ in terms of the observables P and L that have been utilized to compute one of the three constants, given the other two.

A second consistent system of constants has been presented by Clemence (Ref. 6). The major contribution from this paper is a precise statement of the proposed introduction of "ephemeris time." As a consequence of this change (enacted in 1950), several of the inconsistencies of the ephemeris were removed. A precise discussion of this point can be found in the *Explanatory Supplement to the Ephemeris.*

A second conference on the system of constants was held in Paris in 1950 (see Ref. 7). The recommendation of that conference was that the system of constants should not be changed but that the concept of ephemeris time should be made official.

The most current revision of the constants has been given by Brouwer and Clemence (Ref. 8), who have applied primarily current observations to the system developed by de Sitter.

III. RABE'S WORK ON EROS OBSERVATIONS

Rabe (Ref. 9) utilized the observations of Eros at three Earth passings to compute the solar parallax, the Earth–Moon mass ratio, and several other planetary masses as well as corrections to the elements of the Earth. In his computation, the observations of 1930–31 were most heavily weighted. The actual procedure used was to compute the mass of the Earth from the perturbations by the Earth on Eros. Once the mass of the Earth was obtained, the solar parallax was computed from Eq. (9) using de Sitter's constants. The results of Rabe that are of interest here are:

1. $\pi_\odot = 8\overset{''}{.}79835 \pm 0\overset{''}{.}00039$
2. $\mu^{-1} = 81.375 \pm 0.026$
3. $m = 328{,}452 \pm 43$
4. $m_\oplus = 332{,}480$ (from 2 and 3)

It would be possible to revise Rabe's $\pi_\odot$ using slightly different values in de Sitter's Eq. (9), but this would not be profitable (see Ref. 10). However, it has apparently gone unnoticed until recently that the corrections to the elements of the Earth resulting from Rabe's computations are very different from a similar set computed by Duncombe (Ref. 11) from the observations of Venus. It appears likely that if Duncombe's corrections were employed in a new solution for Rabe's normal equations, a significantly different value of the solar parallax might result. The need for such a revision of the Eros results is clear from the strength of the radar results reported below.

IV. VELOCITY OF LIGHT

The determinations of the velocity of light have a long and interesting history. An excellent survey of the classical determinations has been made by Bergstrand (Ref. 12). The adopted value of c as given in the Nautical Ephemeris is a very old determination by Newcomb and is well known to be grossly in error. A precise value of the velocity of light has not been of particular concern in astronomical questions until the present time. The radar determinations of the astronomical unit and the determination of associated constants by radar and radio tracking of artificial space vehicles are intimately concerned with a precise measurement of the velocity of light, however. It will be shown that, even though the modern value of c is known reliably to six figures, the uncertainty in the light-velocity determinations is the major single source of error in the radar measurements when expressed in terms of kilometers.

A recent survey of the important light-velocity determinations since 1946 has been made by DuMond (Ref. 13). His results are shown in Table 1. The best single determination is apparently the value found by Froome (Ref. 14) of

$$299{,}792.50 \pm 0.10 \text{ km/sec}$$

which he obtained by a microwave interferometer technique at 74,500 Mc. The author has computed the mean value from Table 1, weighting the values with the reciprocal squares of the quoted uncertainties, and found a value of

$$299{,}792.63 \pm 0.08 \text{ km/sec}$$

Table 1. Modern velocity of light determinations[a]

Reference	Method	c, km/sec	
18	Shoran	299 792	±3.5
19	Cavity resonance	299 789.3	±1.3
20	Cavity resonance	299 792.5	±1.0
21	Geodimeter	299 793.1	±0.32
22	Microwave interferometer	299 792.6	±0.7
23	Geodimeter	299 792.4	±0.4
24	Microwave interferometer	299 792.7	±0.3
25	Infrared spectrometer	299 792	±6
26	Microwave interferometer	299 795.1	±1.9
12	Geodimeter	299 792.8	±0.34
12	Geodimeter survey	299 792.85	±0.16
14	Microwave	299 792.50	±0.10

[a] Reference 13.

This result is in excellent accord with Froome's individual measurement, partially because of the large weight assigned to Froome's 1958 determination. The general agreement to a few parts in 10^6 of all of the modern values shown in Table 1 is reassuring, and it appears highly unlikely that a systematic error larger than 0.3 km/sec could exist.

The International Union of Geodesy and Geophysics, on the recommendation of the XII General Assembly of the International Scientific Radio Union, has adopted the value of

$$299{,}792.5 \pm 0.4 \text{ km/sec}$$

This value has been used in the radar determinations of the AU.

V. DETERMINATION OF THE AU BY RADAR AT THE 1961 INFERIOR CONJUNCTION OF VENUS

Radar observations have been obtained for Venus around the 1961 inferior conjunction by several groups. The resulting values for the astronomical unit are shown in Table 2. All the determinations are in agreement. However, Newcomb's tables of the Sun and Venus were employed in all cases, which, if they do cause an important error at all, would affect each determination in essentially the same way. A detailed discussion of these effects is presented below.

Table 2. AU determinations from radar observations of Venus[a]

Good radar methods[b]	AU, km	$\pi_{\odot}$, sec
D. Muhleman, *et al.*	149,598,640±250	8.7941379±0.000015
G. Pettingill, *et al.*	149,597,850±400	8.7941849±0.000026
D. Muhleman (revision of Pettingill's value)	149,598,100±400	8.7941705±0.000026
Marginal radar methods		
Thompson, *et al.* (GB)	149,601,000±5000	8.7940±0.003
Maron, *et al.* (USA)	149,596,000	8.7943
Kotel'nikov (USSR)	149,599,500±800	8.7941±0.00005

[a] Reference 2.
[b] Those that observed Venus over a sufficiently long arc to remove the major part of the errors from the ephemerides.

A. Instrumentation

Details of the computations of Muhleman, *et al.*, (Ref. 1) are described in this Section. A complete discussion of Pettingill's result can be found in Ref. 15. The observations reported in the latter paper have been used to compute a slightly revised value of the AU. The observations of Muhleman, *et al.*, were made at the Goldstone station of the Jet Propulsion Laboratory, with three fundamentally different radar receiving systems. The observations consisted of the doppler-frequency shift on the 2388-Mc carrier and measurements of the propagation time to Venus and back to Earth by modulating the carrier with either a regular square wave or a pseudorandom code.

The frequency reference for the doppler velocity measurements was an Atomichron cesium-resonance line which had a measured stability of 1 or 2 parts in 10^{10} over a period of about 5 min. All other reference frequencies in the receiver were coherently derived from the standard in such a manner that frequency errors introduced into the system were subsequently subtracted out at some other point in the closed-loop system. Consequently, the measurements of the doppler frequency shift are probably accurate to better than 1 part in 10^7. This uncertainty is far smaller than that due to the velocity of light.

The systems of modulation employed by the two methods of measuring the propagation time were designed to have a range resolution of about 100 km. The over-all accuracies of this system are on the order of 100 km except for the uncertainty of c, i.e., about 0.0003 sec for the Earth–Venus distance.

B. Preparation of the Ephemeris

The doppler frequency shift and the propagation time must be computed from the ephemerides with precision for comparison with observations. The total propagation time is given by

1. the time for the signal to travel from the position of the transmitting antenna at time 1 to the *surface* of Venus at time 2,

2. plus the time for the signal to travel from the surface of Venus at time 2 to the position of the receiving antenna at time 3.

The actual epoch for each observation was taken to be time 3, and the arguments for entries into the tables of the Sun and Venus were computed with a simple iteration scheme. The doppler-frequency shift is a function of

1. the velocity of the *center of mass* of Venus at the instant the wave front strikes the surface of the planet, with respect to the position and velocity of the transmitting station at time 1, $\dot{R}_{12}$,

2. the velocity and position of the receiving station at the instant the reflected wave front reaches the receiving station, with respect to the velocity of the center of mass of Venus at the instant of reflection, time 2, $\dot{R}_{23}$.

The equation for the conversion of the ephemeris velocities $\dot{R}_{12}$ and $\dot{R}_{23}$ to doppler frequency shift has been derived by Muhleman (Ref. 1) to the second order in v/c

and is

$$(\tilde{\nu} - \nu) = -\nu\left(\frac{\dot{R}_{12}}{C} + \frac{\dot{R}_{23}}{C} - \frac{\dot{R}_{12}\dot{R}_{23}}{C^2} - \frac{\dot{R}^2_{23}}{C^2}\right) \quad (12)$$

where ν is transmitter frequency and $\tilde{\nu}$ is the received frequency at time 3.

The actual values used in the analysis of the radar observations were computed with a tracking program written for the IBM 7090 computer. The coordinates to be smoothed were obtained directly from Newcomb's tables of the Sun and Venus, with corrections for known errors. In particular, a correction of $-4''78T$ was applied to the mean anomaly of the Sun after Clemence (Ref. 6). An n-body numerical integration, starting with injection position and velocity, was compared with the coordinates written on a magnetic tape from the Newcomb tables, and corrections to the injection conditions were derived using a least-squares iterative procedure. Several iterations yielded the best injection values over a 120-day arc for Venus and a 70-day arc for the Earth. These residuals were reduced to a few parts in 10^7 which is consistent with the roundoff in the tabulated data. Velocity data were obtained at each epoch of interest as a consequence of the Runge–Kutta numerical integration procedure. The velocities obtained in this manner are smooth to seven figures and probably accurate to a few parts in 10^6. The ephemerides obtained with the above technique are considered a smooth equivalent to the numerical tables of Newcomb, including only the change in the argument M referred to above. Subsequently in this Report, the ephemerides will be referred to as the Newcomb ephemerides.

Duncombe (Ref. 11) has obtained a set of corrections to Newcomb's elements from the Venus observations over a period from 1795 to 1949. The published corrections are:

for Earth

$$\begin{aligned}
\Delta e_{\oplus} &= -0''10 \pm 0''01 + 0''00\ T \\
\Delta \epsilon &= +0''04 \pm 0''01 - (0''29 \pm 0''03)\ T \\
\Delta L_{\oplus} &= -0''39 \pm 0''05 + (0''45 \pm 0''15)\ T \\
e\Delta \pi &= -0''07 \pm 0''03 - 0''09\ T
\end{aligned}$$

for Venus

$$\begin{aligned}
\Delta l_{♀} &= +0''10 \pm 0''06 + (0''53 \pm 0''18)\ T \\
\Delta e_{♀} &= -0''12 \pm 0''03 + 0''01\ T \\
e_{♀}\Delta \pi_{♀} &= +0''01 \pm 0''04 + 0''04\ T \\
\Delta i_{♀} &= +0''08 \pm 0''03 - 0''02\ T \\
\sin_{♀} \Delta \Omega_{♀} &= +0''21 \pm 0''03 + 0''02\ T
\end{aligned}$$

The corrections actually used were supplied by Duncombe (Ref. 16) and are only slightly different:

for the Earth

$$\begin{aligned}
\Delta e_{\oplus} &= -0''113\ T \\
\Delta \epsilon &= +0''045 - 0''29\ T \\
\Delta M_{\oplus} &= +4''78\ T \text{ (already applied in the Newcomb ephemerides)}
\end{aligned}$$

for Venus

same as above

The Duncombe corrections were incorporated into the program which evaluated the Newcomb theory, and a new ephemeris was generated utilizing the same technique as before. This ephemeris has been called the Duncombe ephemeris.

C. Results

Observations of Venus were made at 10-sec intervals over continuous periods of from 5 min to 1 hr. This was normally done daily for the doppler measurements and the two ranging-systems measurements. Each set of observations was used to compute a separate estimate of the AU, which was computed with an iterative least-squares procedure, which minimized the observations minus the calculated value by computing a correction to the AU value used in the previous iteration. The calculations were performed for both the Newcomb ephemeris and the Duncombe ephemeris. The rms residuals for the velocity observations were about ± 0.1 m/sec, and about ± 200 km was obtained for the range residuals. Actually, the residuals varied somewhat with the distance to Venus because of the decrease in the radar-echo power with distance.

The computed AU estimates from the velocity observations are shown in Fig. 1. This Figure shows that the estimates of the AU rapidly diverge downward as conjunction (April 11) is approached from the east and return from above immediately after conjunction has passed. The effect of the Duncombe corrections was to raise the estimates on March 23 by 1200 km and on April 7 by about 7000 km. Similarly, on April 13 the estimate was lower by 8900 km and on May 3, by 400 km. Clearly, the effect is due to the sensitivity of the doppler velocity (range rate) to errors in the ephemerides as the velocity becomes small. The primary correction of Duncombe is to advance the longitude of Venus by about 0''55 relative to that of the Earth. This was apparently not enough to straighten the curve completely. Muhleman, *et al.*, (Ref. 1) have shown that the effect of an error in the longitudes of Venus and the Earth in the determination of the AU is approximately (near conjunction)

$$\delta(\mathrm{AU}) \simeq A_{\oplus} \cot (l_{♀} - l_{\oplus})\, \delta\, (l_{♀} - l_{\oplus}) \qquad (13)$$

which is very similar to the behavior shown in Fig. 1. A more exact analysis of this problem will be given below.

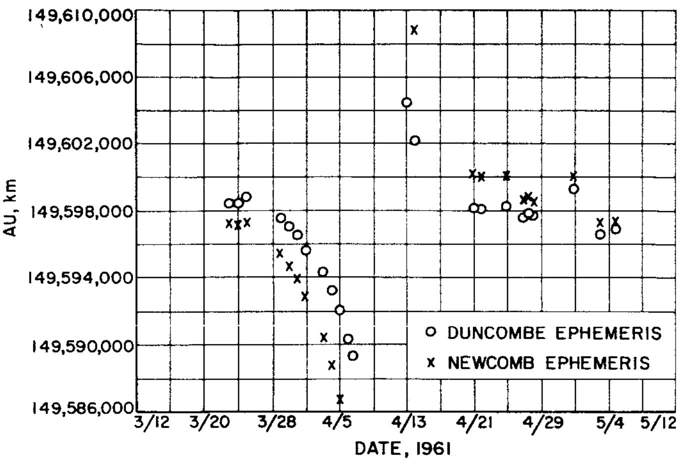

Fig. 1. AU computed from Goldstone velocity observations

The estimates of the AU computed from the range measurements from the system employing the pseudorandom code modulation are shown in Fig. 2. These observations are all postconjunction. A linear trend with date is evident from the Figure, the slope of which was decreased by applying the Duncombe corrections. Muhleman, *et al.*, (Ref. 1) have shown that the effect on the the AU determinations from range data of only an error in the relative planetary longitudes is approximately

$$\delta(\mathrm{AU}) \simeq A_{\oplus} \left(\frac{r_{♀} r_{\oplus}}{r^2}\right) \sin\, (l_{♀} - l_{\oplus})\, \delta\, (l_{♀} - l_{\oplus}) \qquad (14)$$

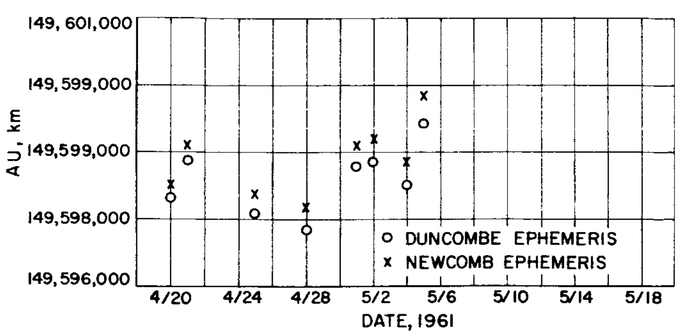

Fig. 2. AU computed from Goldstone range observations

where $r_{♀}$ and $r_{\oplus}$ are the heliocentric distances to the planets and r is the distance between them. The equation agrees well with the effect observed in Fig. 2.

The measured radar propagation times to Venus published by Pettingill, *et al.*, (Ref. 15) were used to compute the estimates of the AU shown in Fig. 3. The agreement between these estimates and those computed by Pettingill is excellent. A trend similar to that predicted by Eq. (14) is again evident in the estimates.

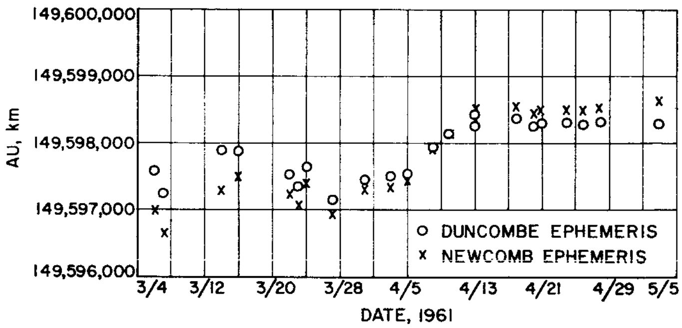

Fig. 3. AU computed from Millstone observations

The reduction of all of the AU estimates to a single result is a considerable task. Because of the apparent errors in the ephemerides (after Duncombe's corrections), it is necessary to proceed somewhat arbitrarily. Equation (13) was used to extrapolate the doppler-AU estimates toward the east and west elongations, where errors in longitude would have a minimal effect. However, an error in $e''\Delta\pi''$ may be significant at these points. Equation (14) was employed to interpolate the range-AU estimates at conjunction. (Clearly, the total effect of the Duncombe corrections is nearly zero at conjunction.)

The results of this procedure are:

1. Doppler near eastern conjunction 149,598,750 ±200 km
2. Doppler near western elongation 149,598,000 ±1000 km
3. Range at conjunction 149,598,500 ±150 km
4. Range at conjunction 149,598,800 ±150 km

where the value in (4) was computed from range observations from the second ranging system, which was independent of the first system to a large degree. The uncertainties attached to the above values are estimates based primarily on the scattering in the estimates. The systematic errors will be considered below.

The final value of the AU is the mean of the four figures above, with weights equal to the reciprocal variances:

$$149{,}598{,}640 \pm 200 \text{ km}$$

The value computed from Pettingill's observations utilizing Eq. (14) for interpolation to conjunction is

$$149{,}598{,}100 \pm 400 \text{ km}$$

where the uncertainty was taken from Pettingill, *et al.*, (Ref. 15).

VI. DETERMINATION OF THE AU BY RADAR AT THE 1962 INFERIOR CONJUNCTION OF VENUS

The observational program on Venus for 1961 was repeated around the 1962 inferior conjunction. The techniques that were employed in the latter observations were somewhat different. In 1961, two antennas separated by 10 km were operated as a transmitter and receiver pair and consequently yielded continuous runs of data. However, it was necessary to use a single antenna in 1962 as both the transmitter and the receiver. This was done by transmitting for the propagation time from the Earth to Venus and switching to the receiver mode for a similar length of time. This reduced the observation time by one-half. Furthermore, it was decided that a comparison ephemeris should be constructed over an arc much longer than the 100-day arcs utilized in the previous analysis in order to cover both observational periods with one fit. The ephemeris was prepared in essentially the manner described above, but 10-year arcs were employed as reported by Peabody and Block (Ref. 17). The residuals in positions relative to the Newcomb tables (after a correction of $M'' = +4''.78\,T$) exhibited oscillations as large as 5×10^{-7} AU in the radius vectors and $0''.1$ in the longitudes and latitudes with the sidereal periods. These residuals have had serious effects on the AU results. Primarily for this reason, the 1962 results reported here are to be considered as preliminary. However, in all cases the values of the AU deduced agree to within the accuracy of the analysis with those found in 1961.

A. Calculation of the Astronomical Unit

The AU has been obtained by comparing the observations to the values computed from the astronomical tables, using a first guess of the AU for entry into the tables and then computing a second estimate of the AU from the differences by the classical least-squares technique. The process is repeated until the rms differences (residuals) obtained in the nth iteration are not significantly smaller than those obtained in the $(n-1)$th iteration. Thus, the AU is found by assuming that the astronomical tables are correct except for one parameter, the AU. In general, a given residual is given by (after a Taylor's expansion to first order)

$$(R_O - R_c)_i = \left(\frac{\partial R_c}{\partial \alpha_1}\right)_i \delta\alpha_1 + \left(\frac{\partial R_c}{\partial \alpha_2}\right)_i \delta\alpha_2 + \cdots + \left(\frac{\partial R_c}{\partial \alpha_m}\right)_i \delta\alpha_n \quad (15)$$

where R_O is the observed range (for example) and R_c is the range computed from the tables with an assumed value of the AU. The $\delta\alpha$ are the (unknown) errors in the significant parameters of the planetary theory *including* the AU. Thus, the method employed here assumes that all of the $\delta\alpha$ are zero except δAU. When the set of equations (Eqs. 15, the normal equations) are solved in a least-squares sense, the resulting correction for the AU in the case in which all of the other $\delta\alpha$ are zero is

$$\delta\text{AU} = \frac{-\sum_i \left(\frac{\partial R_c}{\partial \text{AU}}\right)_i (R_M - R_c)_i}{\sum_i \left(\frac{\partial R_c}{\partial \text{AU}}\right)_i^2} \quad (16)$$

A similar expression can be written for δAU for the doppler observations. The solution for a general set of $\delta\alpha$ merely involves an inversion of the matrix of coefficient from Eq. (15).

A total of 52 doppler runs was made over the period from October 11 to December 17, 1962. The average number of samples per run was 141, and the *average* standard deviation of the final residuals for each run was 2.54 cps. The actual standard deviations are a function of signal-to-noise ratio, and they vary from about 3.5 cps at the beginning and end of the observational period to about 1.2 cps at the time of conjunction. Clearly, the uncertainty in a given estimate of the AU from any single run depends further on the total doppler shift at that time and is widely variable. At the points of greatest interest in the case of the doppler, i.e., the farthest from conjunction where the doppler shift is the greatest, the following uncertainties in the AU have been computed, *based entirely on the above internal statistics*, assuming no correlation between samples:

October 21	$\sigma_{\mathrm{AU}} = 195\,\mathrm{km}$
December 12	$\sigma_{\mathrm{AU}} = 209\,\mathrm{km}$

The resulting estimates of the AU using the Newcomb ephemerides are shown in Figs. 4 and 5.

A total of ten estimates of the AU have been made from the range data over a period from November 8 to December 15, 1962. The average number of samples per run was 472, and the average standard deviation was 614 μsec round-trip propagation time. However, the range residuals are highly correlated. If it is assumed that the residuals are correlated over, say, 25 points, the average run has an uncertainty of 614 times the square-root of 472/25 or 141 μsec, which corresponds to 42.3 km in round-trip range. Adopting this value for the range uncertainty for a measurement at conjunction gives 79 km in the AU *based on these statistics alone*. The resulting estimates of the AU are shown in Fig. 6.

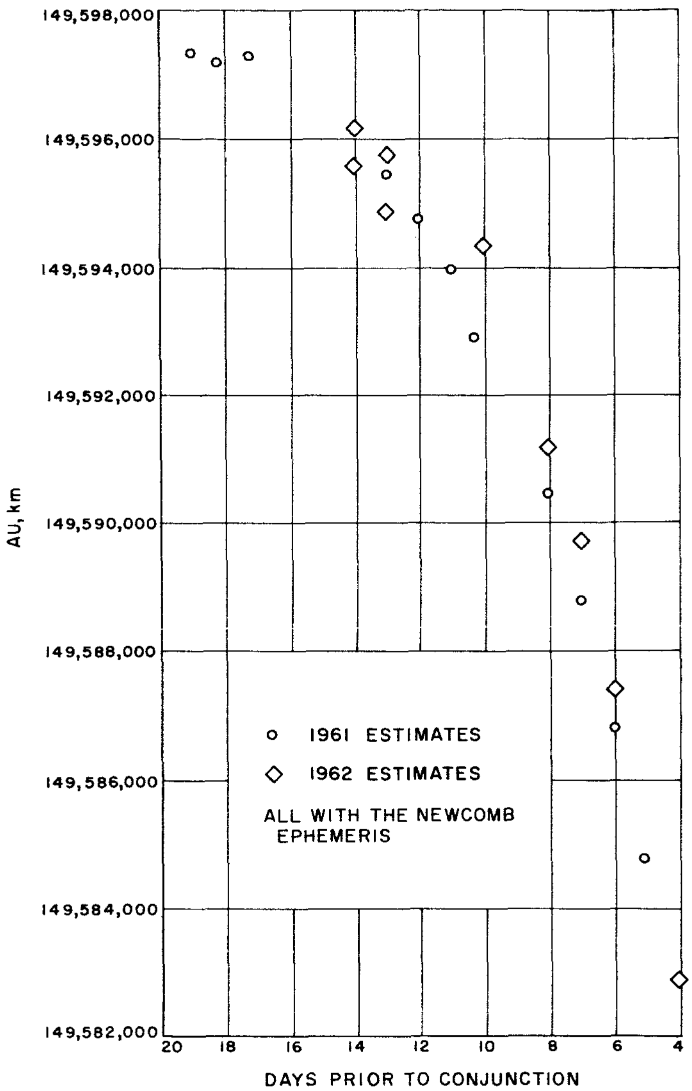

Fig. 4. Comparison between 1961 and 1962 doppler velocity AU's

B. Range and Doppler AU Results

The doppler AU results shown in Figs. 4 and 5 exhibit exactly the same variation with date as those reported by Muhleman, *et al.*, (Ref. 1) for 1961. It is certain that this variation is due to errors in the orbital elements of the Earth and Venus employed in Newcomb's tables. In particular, small changes in the mean longitudes and/or the perihelia of the Earth and Venus would essentially remove this variation.

A Duncombe ephemeris for the 1962 observations has not been computed as yet. Consequently, it was necessary to compute analytically the change in the AU estimate resulting from the Duncombe corrections at each point of interest. It turns out that the effect of the corrections is smallest at specific times in the observational period, i.e., at the points furthest from conjunction for the doppler data and the point at conjunction for the range data. Since these points are the least sensitive to the corrections, they are probably the most accurate estimates of the AU, at least for the types of errors considered. The correction procedure follows from Eq. (15). If δc_1 is identified with the correction to the AU, the result, upon solving Eq. (15) for δc_1, is

$$\delta c_1 = \delta \mathrm{AU} = \frac{(R_M - R_O) - \dfrac{\partial R_O}{\partial c_2}\delta c_2 - \cdots - \dfrac{\partial R_O}{\partial c_M}\delta c_M}{\left(\dfrac{\partial R_O}{\partial c_1}\right)_1} \tag{17}$$

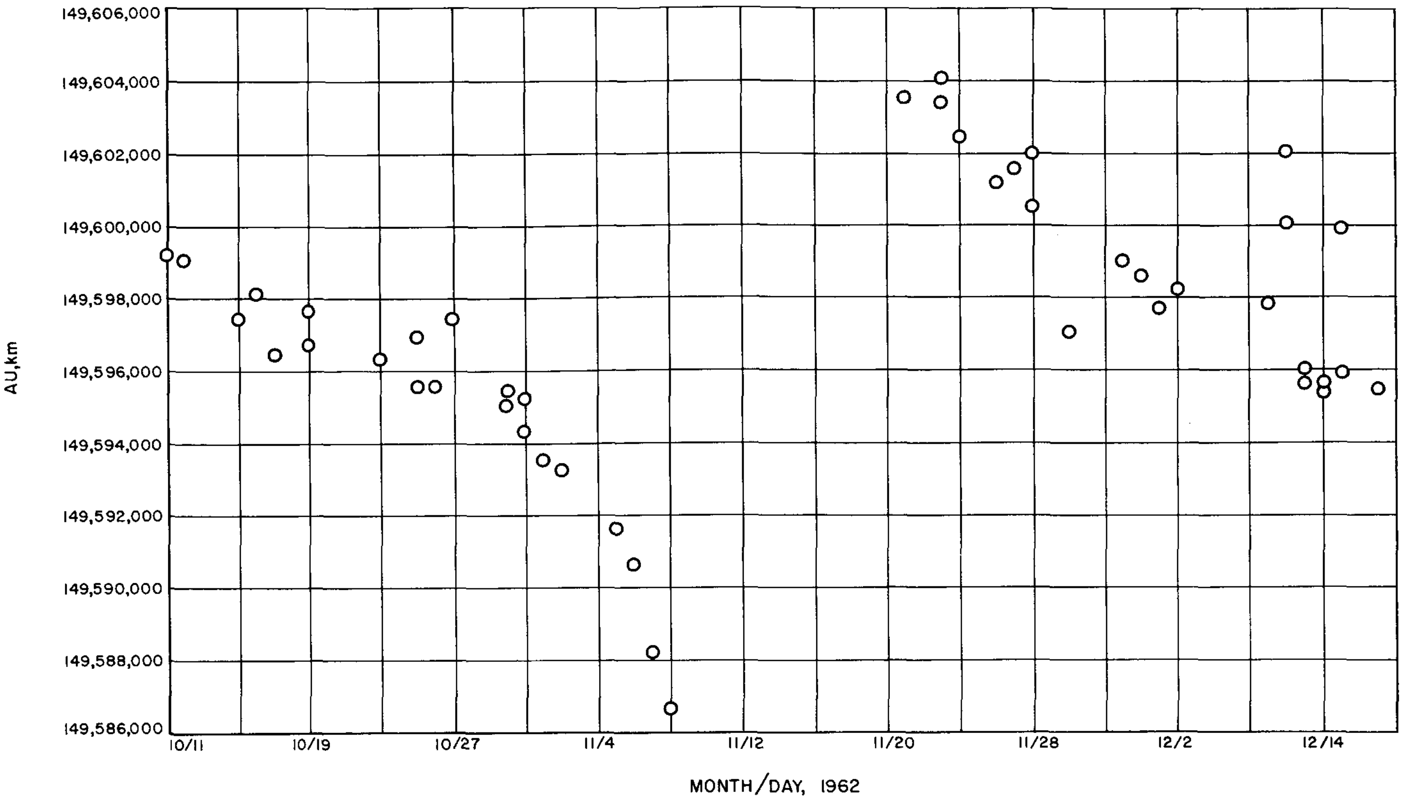

Fig. 5. AU estimates from 1962 doppler observations using the Newcomb ephemerides

But the term $(R_M - R_O)$ has been iterated to zero. Therefore,

$$\delta \mathrm{AU} = \frac{-\dfrac{\partial R_O}{\partial c_2}\delta c_2 - \cdots - \dfrac{\partial R_O}{\partial c_M}\delta c_M}{\dfrac{\partial R_O}{\partial \mathrm{AU}}} \tag{18}$$

where $\delta c_2 = \Delta L''$, $\delta c_3 = \Delta e''$, etc. The partial derivatives in (18) have been computed from analytical expressions utilizing a digital computer program. An expression similar to (18) can be written for the doppler data. The individual terms in δAU are shown in Table 3 for the range observation of November 12, 1962. The actual AU estimates listed in Table 4 were obtained by computing the weighted mean of the estimates near the date of interest. It is clear from Table 4, as well as Fig. 5, that the value for December 12 is anomalously low. A similar effect but of much smaller magnitude was observed one month after conjunction in 1961. Figure 5 suggests that the observations in this region may have been faulty, but no explanation can be offered to support this conjecture. Some insight can be gained by the following analysis, however.

Table 3. Effect of the Duncombe corrections on the AU

	Range Nov. 12
$\Delta L''$	-4
$\Delta e''$	-191
$e''\Delta\pi''$	$+319$
ΔI	-1
Δe	-40
$e\Delta\pi$	-45
Δp	-67
Δq	-8
Totals	-37

Table 4. 1962 AU results

	Newcomb ephemerides[a]	Duncombe ephemerides[b]
Doppler, October 12	149,599,060 km	—
Range, November 12	149,599,730 km	149,599,374 km
Doppler, December 12	149,596,452 km	—

[a] Newcomb ephemerides means Newcomb's tables with a mean anomaly correction of $\Delta M_\oplus = +4\overset{''}{.}78\ T$.
[b] Duncombe ephemerides means here that *only* $\Delta e''$ has been applied for the Earth plus all of the Venus corrections.

The true longitude of the Sun, λ, is computed from Newcomb's tables using the equation

$$\lambda = L'' - (f'' - M'') + \text{perturbation terms} \tag{19}$$

where f'' and M'' are the true anomaly and mean anomaly of the Sun, respectively. To first order in e'',

$$f'' - M'' = 2e'' \sin M'' \tag{20}$$

Then, from (19),

$$\lambda = L'' - 2e'' \sin M + \text{perturbation terms} \tag{21}$$

Now the only change made to Newcomb's tables was $\Delta M'' = -4\overset{''}{.}78\ T$. From (21), for a change of M'' only, the result is

$$\Delta\lambda = 2e''\Delta M'' \cos M''$$

There is a slight change in the perturbation terms due to a change in ΔM, but it is negligible. It turns out that $\cos M''$ for October 12 is 0.135, whereas for December 12, it is 0.922. Thus, any change in M'' has about 7 times the effect on the latter date than on the former. Actually, the inclusion of $-4\overset{''}{.}78\ T$ had an effect on the AU estimate for October 12 of +13 km and for December 12 of +111 km. Clearly, it is possible to raise the AU estimate of December 12 by a very large amount without lowering the estimate of October 12 significantly, with a correction to M'' (or $e''\Delta\pi''$). However, an impossibly large $\Delta M''$ is required to bring the two estimates into complete agreement. It may be concluded from this that the ephemeris errors introduced into the AU computations are probably large compared to the uncertainties of the fundamental radar observations. These errors include those in the Newcomb tables, Duncombe corrections to this table, and probably the most significant, errors in our numerical representation of the ephemerides.

C. *Weighted Mean Results and Comparison With Previous Radar Results*

We shall adopt the mean of AU estimates reported in Table 4 weighted by estimated variances based on the noise in Figs. 5 and 6 and estimated ephemeris uncertainties. Adopting

149,599,060 ±1000 km for October 12, 1962
149,599,374 ±1000 km for November 12, 1962
149,596,452 ±2000 km for December 12, 1962

the preliminary 1962 result is

149,598,900 ±670 km

The final AU results from the 1961 observations reported by Muhleman (Refs. 2, 3) are shown in Table 5.

Table 5. 1961 radar results

Doppler near eastern elongation	149,598,750 ± 200 km
Doppler near western elongation	149,598,000 ±1000 km
Range at conjunction (closed loop)	149,598,500 ± 150 km
Range at conjunction (radiometer)	149,598,800 ± 150 km
Millstone result	149,597,850 ± 400 km
Muhleman's rework of Millstone data	149,598,100 ± 400 km
Weighted mean of 1, 2, 3, and 4	149,598,640 ± 200 km

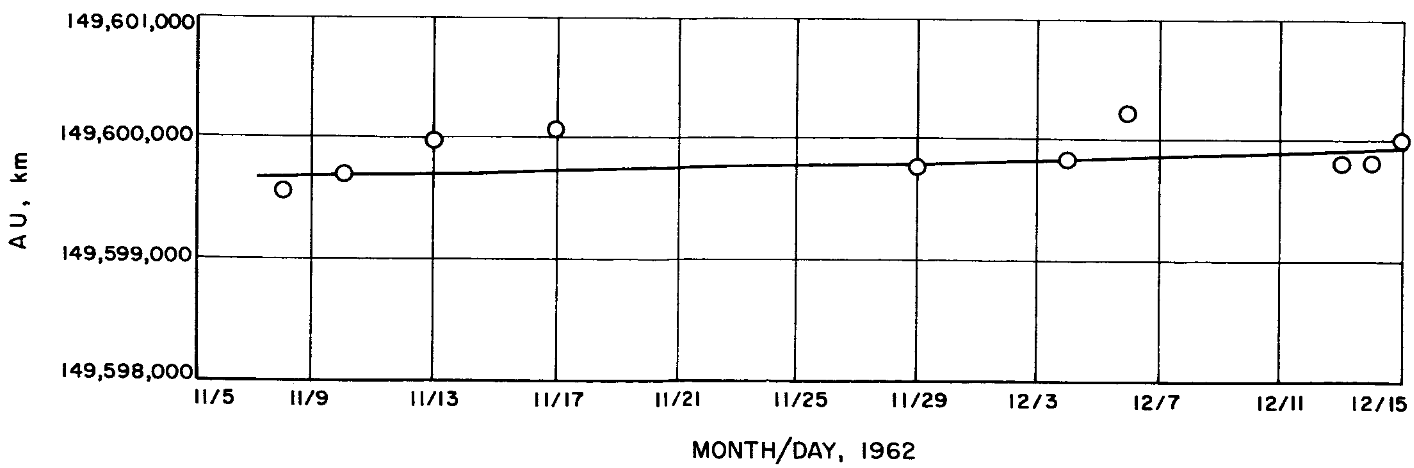

Fig. 6. AU estimates from range observation (Solid line is a weighted linear fit.)

D. Conclusions Concerning the AU

The preliminary best value of the astronomical unit from the observations of Venus around the 1962 inferior conjunction is

$$149{,}598{,}900 \pm 670 \text{ km}$$

where most of the uncertainties are due to the ephemeris errors. This result is in complete agreement with the 1961 Goldstone radar result of

$$149{,}598{,}640 \pm 200 \text{ km}$$

as well as with the results from the 1961 Millstone radar observations.

The remaining uncertainties are linked primarily to the uncertainties in the ephemerides of the Earth and Venus and are of such a nature that the radar observations will ultimately yield definitive corrections to the fundamental ephemerides. This ultimate result is difficult to obtain from an analytical standpoint and will evolve slowly. While it is clear that the observations available at this time are of sufficient quality and quantity to accomplish a good measure of this goal, it should be realized that observations distant from conjunction are required to solve for certain of the corrections that are strongly correlated. In particular, radar observations from the Earth on other planets (or asteroids) are highly desirable for the separation of the effects of the Earth's orbit from those of the orbit of Venus.

VII. ERROR ANALYSIS

A. Velocity of Light

The uncertainty in the vacuum velocity of light was shown to be ± 0.3 km/sec; this appears pessimistic. The effect on the radar values of the AU is then approximately $\pm 0.3 \times 500$ sec or 150 km.

B. Dispersion and Refraction

The effects of signal delays and refraction in the Earth's atmosphere are completely negligible at the frequencies of operation utilized by the Goldstone group (2300 Mc) and Pettingill (440 Mc). The effect of refraction in the atmosphere of Venus is probably negligible because the echo power primarily passes through the Venusian atmosphere at normal incidence.

The question of possible delays in the Venusian atmosphere is much more complex, however. An exhaustive discussion of the point is presented in Refs. 2 and 3. Briefly, the effect of any delay in the atmosphere is to make the propagation time longer than that for the vacuum case and hence, cause the determined value of the AU to be larger. Furthermore, according to the modern theories of propagation, any delaying medium would have an effect increasing with decreasing frequency; thus, the value of the AU determined by radar at 440 Mc should be larger than that computed from observations at 2300 Mc. In fact, it has been shown that if the value of the AU from the 2300-Mc observations is in error by 100 km, then the value measured at 440 Mc should be larger by about 7000 km, whereas the value determined above is actually smaller at 440 Mc by 540 km than the value at 2300 Mc. Thus, it is unlikely that there is any delay effect at all.

C. The Radius of Venus

The uncertainty in the radius of Venus does not affect the value of the AU determined from the doppler frequency. The effect on the range measurements is equal to the radius uncertainty. If the uncertainty of the Venusian radius is taken to be 25 km, the effect on the AU is about 89 km.

D. The Ephemerides

The only reasonable estimate of the ephemeris errors is the Duncombe corrections themselves. It is difficult to see how the errors in the ephemerides *after* corrections could be as large as the corrections themselves. Consequently, Duncombe's values can logically be taken as upper bounds on the errors; but this appears too pessimistic. Therefore, it is desirable first to analyze the range case, and second, the doppler observations.

The range between Venus and the Earth, r, given by

$$r^2 = r_♀^2 + r_⊕^2 - 2r_♀ r_⊕ \cos\theta \tag{22}$$

where $r_♀$ and $r_⊕$ are the solar distances to the planets and θ is the heliocentric angle between the Earth and Venus given by

$$\cos\theta = \cos(l_♀ - \Omega_♀)\cos(l_⊕ - \Omega_♀) + \sin(l_♀ - \Omega_♀)\sin(l_⊕ - \Omega_♀)\cos i_♀ \tag{23}$$

Thus, r is a function of the eccentricities and the arguments of the perihelia through Eq. (22) and the equations of elliptical motion, and r is a function of $l_♀$, $l_⊕$, $\Omega_♀$ and $i_♀$ through Eq. (23). The uncertainty in the obliquity is neglected because its effect on r is very small. Therefore,

$$r = r(l_⊕, l_♀, e_⊕, e_♀, \pi_⊕, \pi_♀, \Omega_♀, i_♀) \tag{24}$$

where it is assumed that

$$r_⊕ = \frac{a_⊕(1 - e_⊕^2)}{1 + e_⊕\cos(l_⊕ - \pi_⊕)} \tag{25}$$

The quantities $a_⊕$ and $a_♀$ will be assumed precisely known in astronomical units. Then, from Eq. (22),

$$r\,dr = r_♀\left(\frac{\partial r_♀}{\partial e_♀}de_♀ + \cdots\right) + r_⊕\left(\frac{\partial r_⊕}{\partial e_⊕}de_⊕ + \cdots\right) + \text{etc.} \tag{26}$$

All of the partial derivatives are then computed from Eqs. (23) and (25). Now, the error in the AU due to an error dr is

$$\delta(\mathrm{AU}) = A_⊕\frac{dr}{r} \tag{27}$$

where $A_⊕$ is the value of the AU in km. The expression for δ(AU) may then be written for small errors in the elements utilizing the partials. Since the primary interest is in the value of δ(AU) at the 1961 inferior conjunction of Venus, the general expression will be given with all of the expressions evaluated at that epoch. The result is

$$\begin{aligned}\delta(\mathrm{AU}) = 9680\ \mathrm{km}\ \{&0.031\,de_⊕ + 0.0047\,dl_⊕\\ &- 0.276\,e_⊕ d\pi_⊕ + -0.028\,de_♀\\ &+ 0.0014\,dl_♀ - 0.198\,e_♀ d\pi_♀\\ &- 0.029\,di_♀ - 0.13\,d\Omega_♀\}\end{aligned} \tag{28}$$

With the Duncombe corrections inserted for the differentials,

$$\delta(\mathrm{AU}) = \{-30 - 5 + 322 + 33 + 6 + 19 + 19 - 47.4\}\ \mathrm{km}$$

$$(\mathrm{AU}) = +317\ \mathrm{km}$$

Thus, if the ephemerides are in error after correction by as much as the corrections themselves, the error in the AU from the range observations is about 317 km.

The case for the doppler observations is far more complicated. Since the points of interest in this case are toward the east and west elongations, it can be shown that the terms involving $\sin i_♀$ are negligible to first order, and a first-order analysis can be carried out in two dimensions. Since the analysis has been carried out in the plane of the ecliptic, the effects of the obliquity are also ignored. Then, the range rate (or doppler velocity) is approximately

$$\dot r \simeq V_♀(\sin\alpha_♀ - \gamma_♀\cos\alpha_♀) - V_⊕(\sin\alpha_⊕ + \gamma_⊕\cos\alpha_⊕) \tag{29}$$

where

$V_⊕, V_♀$ = orbital speeds of the Earth and Venus

$\alpha_♀$ = the angle between the Sun and the Earth at Venus (similarly for $\alpha_⊕$)

$\gamma_⊕, \gamma_♀$ = the angles of the Earth and Venus velocity vectors from the perpendicular to the radius vectors in the orbital planes

From well known equations of celestial mechanics, to first order in the eccentricities,

$$V_⊕ \simeq n_⊕ a_⊕[1 + e_⊕\cos(l_⊕ - \pi_⊕)] \tag{30}$$

and

$$\gamma_⊕ \simeq e_⊕\sin(l_⊕ - \pi_⊕) \tag{31}$$

Thus, from Eqs. (25), (29), (30), and (31), $\dot r$ can be expressed in terms of the elements and the partial derivatives taken. The results are too complex to write down profitably, and only the resulting expression for the δ(AU) will be presented, with all of the expressions evaluated at the epoch March 23, 1961, the date of observation nearest the eastern elongation and consequently, the point of greatest interest.

$$d\dot{r} = 35.05\ \text{km/sec}\ \{0.13\, de_{♀} - 1.96\, dl_{♀} - 1.18\, e_{♀} d\pi_{♀}\}$$

$$- 29.8\ \text{km/sec}\ \{-1.0\, de_{\oplus} - 1.34\, dl_{\oplus} - 1.74\, e_{\oplus} d\pi_{\oplus}\}$$

Since

$$\delta(\text{AU}) = \frac{A_{\oplus}}{\dot{r}}\, d\dot{r} \tag{32}$$

inserting the Duncombe corrections,

$$\delta(\text{AU}) = -1350\ \text{km}$$

This value is, of course, very large and probably equally pessimistic. If the uncertainties of the corrections are used, the largest term is due to the uncertainty in the longitude of Venus and is 620 km. It is not possible to combine the individual terms in a meaningful statistical manner because the correlation coefficient between the terms may even approach unity. However, it appears safe to say that the error in the AU from the doppler observations is less than 620 km. If this circumstance is correct, the doppler value of the AU has been weighted twice as heavily as it should have been in the final reduction to a single result.

VIII. RADAR MEASUREMENTS OF MERCURY

Unequivocal radar contact of Mercury has been accomplished by the Jet Propulsion Laboratory. The observations were made by transmitting a pure continuous wave with the Venus radar equipment. The echo signal was detected by computing the power spectral density of the received signal in a digital computer. The signal spectrum was shifted down near dc by continuously adjusting the receiver local oscillator to the ephemeris doppler frequency plus an offset of about 100 cps. An example of such a spectrum taken by R. Carpenter of JPL is shown in Fig. 7. The ephemeris was prepared in the same way as the Venus ephemeris. The vertical center line in Fig. 7 indicates the frequency about which the observed spectrum would be centered if the ephemeris were perfect and the value used for the AU = 149,598,640 km were correct. The arrows indicate the amount that the spectrum would be shifted for an error in the AU of ±5000 km for the observation date of May 8, 1963.

Some error in the measurement of the center frequency is to be expected resulting from errors in positioning the local oscillator on the order of 1 or 2 cps. Known errors of the ephemerides would have a similar effect. Thus, unless the spectrum in Fig. 7 was positioned fortuitously, the observations yield an excellent verification of the radar value of the AU.

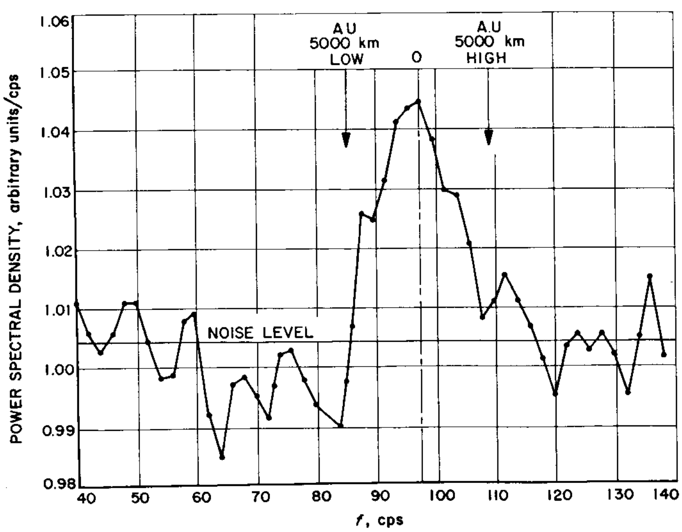

Fig. 7. Spectrum of a Mercury radar echo (Centerline is at frequency at which spectrum would fall if AU = 149,598,640 km.)

Range measurements to Mercury have been accomplished by R. Goldstein of JPL concurrently with the doppler measurements. He has made two measurements, both of which are within about 100 km of the ephemeris values. The ephemeris was computed, of course, using 149,598,640 km for the AU. Doppler measurements of the kind shown in Fig. 7 were made on 10 different days. The differences between the spectral center frequencies and the ephemeris doppler shifts are shown in Fig. 8, where the circles are the measurements of R. Carpenter and the squares are those of R. Goldstein. The solid lines in Fig. 8 represent the error in doppler frequency for an error in the mean anomaly of Mercury of $\Delta M = -2''\!.8$ and an error in the relative mean longitudes of Mercury and the Earth of $1''\!.0$. Therefore, the residuals can easily be explained by the hypothesis of reasonable errors in the Mercury and Earth ephemerides.

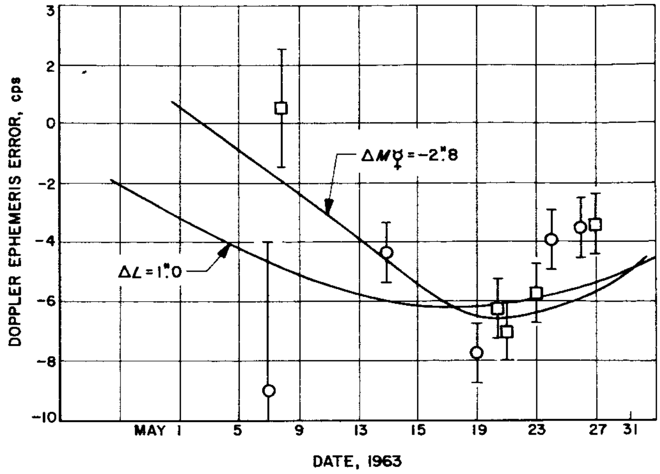

Fig. 8. Observed frequency shift from ephemeris

IX. THE RELATED ASTRONOMICAL CONSTANTS

The relationships presented at the beginning of this Report may now be utilized to construct a consistent set of some of the constants based on the AU result of 149,598,640 ± 250. From Eq. (4), using $b = 6{,}347{,}166$ km, the result for the solar parallax is

$$\pi_\odot = 8''\!.794139 \pm 0''\!.000015$$

The light-time for unit distance is given by Eq. (8):

$$\tau = 499.0073 \pm 0.0007 \text{ sec}$$

It should be realized that τ is the most fundamental result from the radar work because it is independent of the speed of light. The aberration constant is also independent of c when the radar value of the AU is used. From Eq. (7), (4), and (8),

$$K = \frac{n \sec \phi}{86400}\,\tau$$

$$K = 20''\!.49562 \pm 0.00003$$

The Earth–Moon mass ratio can be obtained from the lunar inequality, Eq. (11), which can be written

$$L = \left(\frac{\mu}{1+\mu}\right)\frac{a_☾}{\text{AU}}$$

and the dependence on c is again removed from the radar results if Yaplee's radar value of $a_☾$ is corrected to the same value of c. Using $L = 6.4378 \pm 0.002$ (Brouwer and Clemence, 1961) and $a_☾ = 388{,}400.4$,

$$\mu^{-1} = 81.327 \pm 0.025$$

where the uncertainty is due to that of L.

The coefficient of the parallactic inequality is obtained from Eq. (10), where again c factors out if radar values of $a_☾$ and AU are used:

$$P = +124''\!.987 \pm 0.001$$

Finally, a consistent value of the mass of the Earth plus Moon can be obtained from an expression given by Brouwer (Ref. 10):

$$\frac{S + E + M}{E + M} = 0.0055800307 \frac{(\mathrm{AU})^3}{(a_☾)^3}$$

where Brouwer has obtained the constant term from modern measurements of the Earth constants. Note again that for radar values of AU and $a_☾$, the errors due to c are removed and the result is

$$(E + M)^{-1} = 328{,}903.2$$

The values above cannot be considered definitive until the ephemeris errors are removed from the radar values, but it is clear that all the above constants except $\pi_\odot$ are free from the error in the radar AU introduced by using a specific value of c. Thus, from this standpoint, the major criticism of the radar method, namely the uncertainty of the propagation velocity, is destroyed.

ACKNOWLEDGMENTS

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February 3, 1964

Recipients of Jet Propulsion Laboratory

Technical Report No. 32-477

SUBJECT: Erratum for Technical Report No. 32-477

Gentlemen:

It is requested that the following error be corrected in your copy of Jet Propulsion Laboratory Technical Report No. 32-477, entitled "Relationship Between the System of Astronomical Constants and the Radar Deteminations of the Astronomical Unit," by Duane O. Muhleman, dated January 15, 1964:

Page 5, Table 2, first row, second column: correct 149,498,640 to read 149,598,640.

Very truly yours,

JET PROPULSION LABORATORY

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